

The Effect of Quantum Memory on Quantum Games

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We study quantum games with correlated noise through a generalized quantization scheme. We investigate the effects of memory on quantum games, such as Prisoner's Dilemma, Battle of the Sexes and Chicken, through three prototype quantum-correlated channels. It is shown that the quantum player enjoys an advantage over the classical player for all nine cases considered in this paper for the maximally entangled case. However, the quantum player can also outperform the classical player for subsequent cases that can be noted in the case of the Battle of the Sexes game. It can be seen that the Nash equilibria do not change for all the three games under the effect of memory.

Keywords: quantum games; correlated noise; quantum channels with memory.

I. INTRODUCTION

The study of quantum games combines the laws of quantum mechanics with game theory. It is interesting to study the games at microscopic level where the laws of quantum mechanics dictates the dynamics. Quantum games offer additional strategies to the players and resolve dilemmas that occur in classical games [1–6]. Quantum theory has already been applied to a wide variety of games [7–11] and shown to be experimentally feasible [12]. Additionally, quantum games offer a new paradigm for exploring the fascinating world of quantum information [13–15]. Meyer [16] has also pointed out the connection between quantum games and quantum information processing. In the earlier work on quantum games, for simplicity, the role of channels is mostly ignored. In a realistic setup, however, the flow of information between players and arbiter is subject to interaction with the environment. Quantum entanglement, which is one of the interesting features of quantum mechanics, plays a crucial role in quantum information processing. When quantum information processing is performed in the real world, decoherence caused by an external environment is in-

evitable. In other words, the influence of an external environmental system on the entanglement cannot be ignored. Recently, decoherence effects in quantum games have been studied [17].

Later, interest has been developed to extend the theory of quantum channels to encompass memory effects [18, 19]. There are time scales for which successive uses of channel are correlated and memory effects need to be taken into account. Quantum computing in the presence of noise is possible with the use of decoherence-free subspaces [20] and quantum error correction [21]. Studies concerning quantum games in the presence of decoherence and correlated noise have produced interesting results. Chen et al [17] have shown that in the case of the game Prisoner's Dilemma, the Nash equilibria are not changed by the effects of decoherence for maximally entangled states incorporating three prototype decoherence channels. Recently, Nawaz and Toor [22] have shown for the quantum games based on quantum-correlated phase-damping channel that the quantum player only enjoys an advantage over the classical player when both the initial quantum state and the measurement basis are in entangled form. It is also shown that for maximum correlation the effects of decoherence diminish and it behaves as a noiseless game. Recently, Cao et al [23] have investigated the effect of quantum noise on a multiplayer quantum game. They have shown that in a maximally entangled case a special Nash equilibrium appears for a specific range of quantum noise parameter.

In this paper, we study the quantum games based on three prototype quantum correlated channels (QCC) parameterized by a memory factor μ which measures the degree of correlations, in the context of generalized quantization scheme for non-zero sum games [24]. We identify four different regimes on the basis of initial state and measurement basis entanglement parameters, $\gamma \in [0, \pi/2]$ and $\delta \in [0, \pi/2]$, respectively. For these four regimes, we study the role of decoherence parameter $p \in [0, 1]$ and memory parameter $\mu \in [0, 1]$ for three quantum games. Here, $\delta = 0$ means that the measurement basis are unentangled and $\delta = \pi/2$ means that it is maximally entangled, $\gamma = 0$ means that the game is initially unentangled and $\gamma = \pi/2$ means that it is maximally entangled. Whereas the lower and upper limits of p correspond to a fully coherent and fully decohered system, respectively. Furthermore, the lower and upper limits of μ correspond to a memoryless and maximum memory (degree of correlation) cases, respectively. It is shown that for $\gamma = \delta = 0$, with decoherence and noise parameters $p_1 = p_2 = 0$ and $\mu_1 = \mu_2 = 0$, respectively, the game reduces to the classical one for all the cases discussed in this paper. In Prisoner's Dilemma game, when $\gamma \neq 0$, $\delta = 0$, it is interesting to note that though the initial state is entangled, the quantum player has no advantage over the classical player in Prisoner's Dilemma and Chicken games. The same happens for the case of $\gamma = 0$, $\delta \neq 0$. An interesting aspect of these cases arises

based on entangling parameter γ and measurement parameter δ for $\delta = 0, \gamma \neq 0$ and $\gamma = 0, \delta \neq 0$ in Battle of Sexes game. It is seen that the quantum player is better off for both of the above cases for $p > 0$ in case of amplitude-damping and depolarizing channels respectively. For the case when $\gamma = \delta = \pi/2$, the quantum player always remains better off for all values of p against a player restricted to classical strategies for all the nine cases considered.

II. QUANTUM CHANNELS WITH MEMORY

Several investigations concern the transmission of quantum information from one party (Alice) to another (Bob) through a communication channel. In the most basic configuration the information is encoded in qubits. If the qubits are perfectly protected from environmental influence, Bob receives them in the same state prepared by Alice. In the more realistic case, however, the qubits have a nontrivial dynamics during the transmission because of their interaction with the environment [25]. Therefore, Bob receives a set of distorted qubits because of the disturbing action of the channel. Recently, the study of quantum channels has attracted a lot of attention [18, 19, 26]. Early works in this direction were devoted, mainly, to memoryless channels for which consecutive signal transmissions through the channel are not correlated. Correlated noise, also referred as memory in the literature, acts on consecutive uses of the channels. However in general one may want to encode classical data into entangled strings or, consecutive uses of the channel may be correlated to each other. Hence, we are dealing with a strongly correlated quantum system, the correlation of which results from the memory of the channel itself. In our model Alice and Bob, each uses individual channels to communicate with the arbiter of the game. Alice's channel is correlated in time (and therefore has a memory), i.e. the two uses of the channel; the first passage (from the arbiter) and the second passage (back to the arbiter) through the channel are correlated. A similar situation occurs for Bob as depicted in figure 1. We consider here different noise models based on phase-damping, amplitude-damping and depolarizing channels.

The action of transmission channels is described by Kraus operators which satisfy $\sum_{i=0}^1 A_i^\dagger A_i = 1$. In operator sum representation the dephasing process can be expressed as [25].

$$\rho_f = \sum_{i=0}^1 A_i \rho_{in} A_i^\dagger \quad (1)$$

where ρ_{in} represents the initial density matrix for quantum state and

$$\begin{aligned} A_0 &= \sqrt{1 - \frac{p}{2}} I \\ A_1 &= \sqrt{\frac{p}{2}} \sigma_z \end{aligned} \quad (2)$$

are the Kraus operators, I is the identity operator, p is the decoherence parameter and σ_z is the Pauli matrix. Let N qubits are allowed to pass through such a channel then equation (1) becomes [27]

$$\rho_f = \sum_{k_1, \dots, k_n=0}^1 (A_{k_n} \otimes \dots A_{k_1}) \rho_{in} (A_{k_1}^\dagger \otimes \dots A_{k_n}^\dagger) \quad (3)$$

Now if the noise is correlated with memory of degree μ , then the action of the channel on two consecutive qubits is given by Kraus operator [18]

$$A_{ij} = \sqrt{p_i[(1-\mu)p_j + \mu\delta_{ij}]} \sigma_i \otimes \sigma_j \quad (4)$$

where σ_i and σ_j are usual Pauli matrices with indices i and j running from 0 to 3. The above expression means that with the probability $1 - \mu$ the noise is uncorrelated whereas with probability μ the noise is correlated as illustrated in the below equations. Physically the parameter μ is determined by the relaxation time of the channel when a qubit passes through it. In order to remove correlations, one can wait until the channel has relaxed to its original state before sending the next qubit, however this lowers the rate of information transfer. Thus it is necessary to consider the performance of the channel for arbitrary values of μ to reach a compromise between various factors which determine the final rate of information transfer. Thus in passing through the channel any two consecutive qubits undergo random independent (uncorrelated) errors with probability $1 - \mu$ and identical (correlated) errors with probability μ . This should be the case if the channel has a memory depending on its relaxation time and if we stream the qubits through it. A quantum dephasing channel (Pauli Z channel) with uncorrelated noise (memoryless channel) can be defined as one specified by the following Kraus operators

$$Z_{ij}^u = \sqrt{p_i p_j} \sigma_i \otimes \sigma_j, \quad i, j = 0, 3 \quad (5)$$

and one with correlated noise (channel with memory) by

$$Z_{kk}^c = \sqrt{p_k} \sigma_k \otimes \sigma_k, \quad k = 0, 3 \quad (6)$$

The action of a depolarizing channel with memory can be expressed as

$$\pi \rightarrow \rho = \Phi(\pi) = (1 - \mu) \sum_{i,j=0}^3 D_{ij}^u \pi D_{ij}^{u\dagger} + \mu \sum_{k=0}^3 D_{kk}^c \pi D_{kk}^{c\dagger} \quad (7)$$

where $0 \leq \mu \leq 1$. With probability $1 - \mu$ the noise is uncorrelated and completely specified by the Kraus operators

$$D_{ij}^u = \sqrt{p_i p_j} \sigma_i \otimes \sigma_j, \quad (8)$$

and one with correlated noise (channel with memory) by

$$D_{kk}^c = \sqrt{p_k} \sigma_k \otimes \sigma_k, \quad (9)$$

where $0 \leq p \leq 1$, $p_0 = (1 - p)$, $p_1 = p_2 = p_3 = p/3$. However, we note that a quantum amplitude-damping channel with uncorrelated noise can be defined as one specified by the following Kraus operators:

$$A_{00}^u = A_0 \otimes A_0, \quad A_{01}^u = A_0 \otimes A_1, \quad A_{10}^u = A_1 \otimes A_0, \quad A_{11}^u = A_1 \otimes A_1 \quad (10)$$

$$A_0 = \begin{bmatrix} \cos \chi & 0 \\ 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ \sin \chi & 0 \end{bmatrix} \quad (11)$$

However, the Kraus operators for a quantum amplitude-damping channel with correlated noise are given by Yeo and Skeen [19] as under:

$$A_{00}^c = \begin{bmatrix} \cos \chi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{11}^c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin \chi & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

where, $0 \leq \chi \leq \pi/2$ and is related to decoherence parameter as

$$\begin{aligned} \cos^2 \chi &= 1 - p \\ \sin^2 \chi &= p \end{aligned} \quad (13)$$

It is clear that A_{00}^c cannot be written as a tensor product of two two-by-two matrices. This gives rise to the typical spooky action of the channel: $|01\rangle$ and $|10\rangle$, and any linear combination of them, and $|11\rangle$ will go through the channel undisturbed, but not $|00\rangle$. The action of this non-unital channel is given by

$$\pi \rightarrow \rho = \Phi(\pi) = (1 - \mu) \sum_{i,j=0}^1 A_{ij}^u \pi A_{ij}^{u\dagger} + \mu \sum_{k=0}^1 A_{kk}^c \pi A_{kk}^{c\dagger} \quad (14)$$

The protocol for quantum games in the presence of correlated noise is developed by Nawaz and Toor [22]. We consider that an initial entangled state is prepared by the arbiter and passed on to the players through three prototype quantum correlated channels (as shown in figure 1). i.e. Alice and Bob each uses individual channels to communicate with the arbiter of the game. Alice's channel is correlated in time (and therefore has a memory), i.e. the two uses of the channel are correlated. On receiving the quantum state from the arbiter, the players apply their local operators (strategies) and return it back to arbiter through QCC. Arbiter then performs the measurement and announces their payoffs. Let the game start with the initial quantum state given below,

$$|\psi_{in}\rangle = \cos \frac{\gamma}{2} |00\rangle + i \sin \frac{\gamma}{2} |11\rangle \quad (15)$$

where $0 \leq \gamma \leq \pi/2$ corresponds to entanglement of the initial state. The strategies of the players in the generalized quantization scheme are represented by the unitary operator U_i of the form [24].

$$U_i = \cos \frac{\theta_i}{2} R_i + \sin \frac{\theta_i}{2} P_i \quad (16)$$

where $i = 1$ or 2 and R_i, P_i are the unitary operators defined as

$$\begin{aligned} R_i |0\rangle &= e^{i\alpha_i} |0\rangle, & R_i |1\rangle &= e^{-i\alpha_i} |1\rangle \\ P_i |0\rangle &= e^{i(\frac{\pi}{2}-\beta_i)} |1\rangle, & P_i |1\rangle &= e^{i(\frac{\pi}{2}+\beta_i)} |0\rangle \end{aligned} \quad (17)$$

where $0 \leq \theta \leq \pi$ and $-\pi \leq \alpha, \beta \leq \pi$. Under the generalized quantization scheme with three parameter strategies, the initial state given in equation (15) transforms to

$$\rho_f = (U_1 \otimes U_2) \rho_{in} (U_1 \otimes U_2)^\dagger \quad (18)$$

where $\rho_{in} = |\psi_{in}\rangle \langle \psi_{in}|$ is the density matrix for the quantum state. The operators used by the arbiter to determine the payoff for Alice and Bob are

$$P = \$_{00}P_{00} + \$_{01}P_{01} + \$_{10}P_{10} + \$_{11}P_{11} \quad (19)$$

where

$$\begin{aligned} P_{00} &= |\psi_{00}\rangle \langle \psi_{00}|, & |\psi_{00}\rangle &= \cos \frac{\delta}{2} |00\rangle + i \sin \frac{\delta}{2} |11\rangle \\ P_{11} &= |\psi_{11}\rangle \langle \psi_{11}|, & |\psi_{11}\rangle &= \cos \frac{\delta}{2} |11\rangle + i \sin \frac{\delta}{2} |00\rangle \\ P_{10} &= |\psi_{10}\rangle \langle \psi_{10}|, & |\psi_{10}\rangle &= \cos \frac{\delta}{2} |10\rangle - i \sin \frac{\delta}{2} |01\rangle \\ P_{01} &= |\psi_{01}\rangle \langle \psi_{01}|, & |\psi_{01}\rangle &= \cos \frac{\delta}{2} |01\rangle - i \sin \frac{\delta}{2} |10\rangle \end{aligned} \quad (20)$$

with $0 \leq \delta \leq \pi/2$ and $\$_{ij}$ are the elements of payoff matrix in the i th row and j th column of classical games as given in appendix A. In the generalized quantization scheme for three set of parameters, the players payoffs read

$$\$^A(\theta_i, \alpha_i, \beta_i) = \text{Tr}(P_A \rho_f), \quad \$^B(\theta_i, \alpha_i, \beta_i) = \text{Tr}(P_B \rho_f) \quad (21)$$

where Tr represents the trace of the matrix. Using equations (4)-(9), (14), (19) and (21), the payoffs of the two players, when both channels (first and second) are amplitude-damping, are given by

$$\begin{aligned} \$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta_1^A \$_{00} + \chi_1^A \$_{11} + \Delta_1^A (\$_{01} + \$_{10}) + (\$_{11} - \$_{00}) \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\ & + s_1 s_2 [\eta_2^A \$_{00} + \chi_2^A \$_{11} + \Delta_2^A (\$_{01} + \$_{10}) - (\$_{11} - \$_{00}) \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(10)} \xi \cos 2(\beta_1 + \beta_2)] \\ & + s_1 c_2 [\eta_3^A \$_{00} + \chi_3^A \$_{11} + \Delta_3^A \$_{01} + \Delta_4^A \$_{10} + (\$_{01} - \$_{10}) \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(b)} \xi \cos 2(\alpha_2 - \beta_1)] \\ & + c_1 s_2 [\eta_3^A \$_{00} + \chi_3^A \$_{11} + \Delta_4^A \$_{01} + \Delta_3^A \$_{10} - (\$_{01} - \$_{10}) \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(b)} \xi \cos 2(\alpha_1 - \beta_2)] \\ & - \frac{1}{4} [\sin(\delta) \sin(\theta_1) \sin(\theta_2) \chi_{\mu_2}^{(10)} (\$_{00} - \$_{11}) \Delta_5^A \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)] \\ & - \frac{1}{4} [\sin(\gamma) \sin(\theta_1) \sin(\theta_2) (\eta_4 \$_{00} + \chi_4 \$_{11}) \Delta_5^A \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2)] \\ & - \frac{1}{4} [\sin(\gamma) \sin(\theta_1) \sin(\theta_2) (\$_{01} + \$_{10}) \Delta_6^A \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2)] \\ & + \frac{1}{4} [\sin(\delta) \sin(\theta_1) \sin(\theta_2) \chi_{\mu_2}^{(b)} (\$_{01} - \$_{10}) \Delta_5^A \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)] \end{aligned} \quad (22)$$

The payoffs of the two players, when both channels are depolarizing, are given as

$$\begin{aligned} \$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^D \$_{00} + \chi^D \$_{11} + \Delta^D (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) (\Delta_{\mu_2}^4 - \frac{2}{3} \mu_2 p_2) \Delta_{\mu_1}^3 \xi \cos 2(\alpha_1 + \alpha_2)] \\ & + s_1 s_2 [\chi^D \$_{00} + \eta^D \$_{11} + \Delta^D (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) (\Delta_{\mu_2}^4 + \frac{2}{3} \mu_2 p_2) \Delta_{\mu_1}^3 \xi \cos 2(\beta_1 + \beta_2)] \\ & + s_1 c_2 [\eta^D \$_{10} + \chi^D \$_{01} + \Delta^D (\$_{00} + \$_{11}) + (\$_{01} - \$_{10}) (\Delta_{\mu_2}^4 - \frac{2}{3} \mu_2 p_2) \Delta_{\mu_1}^3 \xi \cos 2(\alpha_2 - \beta_1)] \\ & + c_1 s_2 [\eta^D \$_{01} + \chi^D \$_{10} + \Delta^D (\$_{00} + \$_{11}) - (\$_{01} - \$_{10}) (\Delta_{\mu_2}^4 + \frac{2}{3} \mu_2 p_2) \Delta_{\mu_1}^3 \xi \cos 2(\alpha_1 - \beta_2)] \\ & - (\frac{1}{4} \Delta_{\mu_1}^3 \Delta_{\mu_2}^1 + \frac{1}{2} \Delta_{\mu_1}^3 \Delta_{\mu_2}^2 - \frac{1}{4} \Delta_{\mu_1}^3 \Delta_{\mu_2}^3) (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\ & - (\frac{1}{4} \Delta_{\mu_2}^4 - \frac{1}{6} \mu_2 p_2) (\$_{00} - \$_{11}) \eta_{1DP} \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\ & + (\frac{1}{4} \Delta_{\mu_1}^3 \Delta_{\mu_2}^1 - \frac{1}{2} \Delta_{\mu_1}^3 \Delta_{\mu_2}^2 + \frac{1}{4} \Delta_{\mu_1}^3 \Delta_{\mu_2}^3) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\ & - (\frac{1}{4} \Delta_{\mu_2}^4 - \frac{1}{6} \mu_2 p_2) (\$_{01} - \$_{10}) \eta_{1DP} \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)] \end{aligned} \quad (23)$$

The payoffs of the two players, when both channels are phase-damping, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^P \$_{00} + \chi^P \$_{11} + (\$_{00} - \$_{11}) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\eta^P \$_{11} + \chi^P \$_{00} - (\$_{00} - \$_{11}) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\eta^P \$_{10} + \chi^P \$_{01} + (\$_{10} - \$_{01}) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\eta^P \$_{01} + \chi^P \$_{10} - (\$_{10} - \$_{01}) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_1 - \beta_2)] \\
& + \frac{\mu_p^{(2)}}{4} (\$_{00} - \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + \frac{\mu_p^{(1)}}{4} (-\$_{00} - \$_{11} + \$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + \frac{\mu_p^{(2)}}{4} (\$_{10} - \$_{01}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)
\end{aligned} \tag{24}$$

The payoffs of the two players, when first channel is phase-damping and second channel is amplitude-damping, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta_1^{PA} \$_{00} + \chi_1^{PA} \$_{11} + \Delta_1^{PA} (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) \mu_p^{(1)} \chi_{\mu 2}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\eta_2^{PA} \$_{00} + \chi_2^{PA} \$_{11} + \Delta_2^{PA} (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) \mu_p^{(1)} \chi_{\mu 2}^{(10)} \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\Delta_3^{PA} \$_{01} + \Delta_4^{PA} \$_{10} + \eta_3^{PA} \$_{00} + \chi_3^{PA} \$_{11} + (\$_{01} - \$_{10}) \mu_p^{(1)} \chi_{\mu 2}^{(b)} \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\Delta_4^{PA} \$_{01} + \Delta_3^{PA} \$_{10} + \eta_3^{PA} \$_{00} + \chi_3^{PA} \$_{11} - (\$_{01} - \$_{10}) \mu_p^{(1)} \chi_{\mu 2}^{(b)} \xi \cos 2(\alpha_1 - \beta_2)] \\
& - \frac{1}{4} (\chi_{\mu 2}^{(00)} + \chi_{\mu 2}^{(11)} - 2\chi_{\mu 2}^{(a)}) (\$_{00} - \$_{11}) \mu_p^{(1)} \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + \frac{1}{4} \chi_{\mu 2}^{(10)} (\$_{00} - \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + \frac{1}{4} \mu_p^{(1)} (\chi_{\mu 2}^{(b)} + \chi_{\mu 2}^{(01)}) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + \frac{1}{4} \chi_{\mu 2}^{(b)} (\$_{01} - \$_{10}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)
\end{aligned} \tag{25}$$

The payoffs of the two players, when first channel is amplitude-damping and second channel is

phase-damping, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^{AP} \$_{00} + \chi^{AP} \$_{11} + \Delta^{AP} (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) \mu_p^{(2)} \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\chi^{AP} \$_{00} + \eta^{AP} \$_{11} + \Delta^{AP} (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) \mu_p^{(2)} \chi_{\mu 1}^{(10)} \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\eta^{AP} \$_{10} + \chi^{AP} \$_{01} + \Delta^{AP} (\$_{00} + \$_{11}) + (\$_{01} - \$_{10}) \mu_p^{(2)} \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\eta^{AP} \$_{01} + \chi^{AP} \$_{10} + \Delta^{AP} (\$_{00} + \$_{11}) - (\$_{01} - \$_{10}) \mu_p^{(2)} \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 - \beta_2)] \\
& - \frac{1}{4} \chi_{\mu 1}^{(10)} (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + \frac{1}{4} \mu_p^{(2)} \eta_{1AD} (\$_{00} + \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + \frac{1}{4} \chi_{\mu 1}^{(10)} (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + \frac{1}{4} \mu_p^{(2)} \eta_{1AD} (\$_{01} - \$_{10}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2) \tag{26}
\end{aligned}$$

The payoffs of the two players, when first channel is amplitude-damping and second channel is depolarizing, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^{AD} \$_{00} + \chi^{AD} \$_{11} + \Delta^{AD} (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) (\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\chi^{AD} \$_{00} + \eta^{AD} \$_{11} + \Delta^{AD} (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) (\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 c_2 [\chi^{AD} \$_{01} + \eta^{AD} \$_{10} + \Delta^{AD} (\$_{00} + \$_{11}) + (\$_{01} - \$_{10}) (\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\eta^{AD} \$_{01} + \chi^{AD} \$_{10} + \Delta^{AD} (\$_{00} + \$_{11}) - (\$_{01} - \$_{10}) (\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 - \beta_2)] \\
& - (\frac{1}{4} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^1 - \frac{1}{2} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^2 + \frac{1}{4} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^3) (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + (\frac{1}{4} \Delta_{\mu 2}^4 - \frac{1}{6} \mu_2 p_2) (\$_{00} - \$_{11}) \eta_{1AD} \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + (\frac{1}{4} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^1 - \frac{1}{2} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^2 + \frac{1}{4} \chi_{\mu 1}^{(10)} \Delta_{\mu 2}^3) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + (\frac{1}{4} \Delta_{\mu 2}^4 - \frac{1}{6} \mu_2 p_2) (\$_{01} - \$_{10}) \eta_{1AD} \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2) \tag{27}
\end{aligned}$$

The payoffs of the two players, when first channel is depolarizing and second channel is amplitude-

damping, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta_1^{DA} \$_{00} + \chi_1^{DA} \$_{11} + \Delta_1^{DA} (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) \Delta_{\mu 1}^3 \chi_{\mu 2}^{(10)} \chi_{\mu 1}^{(10)} \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\eta_2^{DA} \$_{00} + \chi_2^{DA} \$_{11} + \Delta_1^{DA} (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) \Delta_{\mu 1}^3 \chi_{\mu 2}^{(10)} \chi_{\mu 1}^{(10)} \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\Delta_2^{DA} \$_{01} + \Delta_3^{DA} \$_{10} + \eta_3^{DA} \$_{00} + \chi_3^{DA} \$_{11} + (\$_{01} - \$_{10}) \Delta_{\mu 1}^3 \chi_{\mu 2}^{(b)} \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\Delta_3^{DA} \$_{01} + \Delta_2^{DA} \$_{10} + \eta_3^{DA} \$_{00} + \chi_3^{DA} \$_{11} - (\$_{01} - \$_{10}) \Delta_{\mu 1}^3 \chi_{\mu 2}^{(b)} \xi \cos 2(\alpha_1 - \beta_2)] \\
& - (\frac{1}{4} \chi_{\mu 2}^{(00)} \Delta_{\mu 1}^3 + \frac{1}{4} \Delta_{\mu 1}^3 + \frac{1}{4} \Delta_{\mu 1}^3 \chi_{\mu 2}^{(11)}) (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& - (\frac{1}{4} \eta_{1DP} \chi_{\mu 2}^{(10)}) (\$_{00} - \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& - (\frac{1}{2} \chi_{\mu 2}^{(01)} \Delta_{\mu 1}^3 - \frac{1}{4} \chi_{\mu 2}^{(b)} \Delta_{\mu 1}^3) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& - (\frac{1}{4} \eta_{1DP} \chi_{\mu 2}^{(b)}) (\$_{01} - \$_{10}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)] \quad (28)
\end{aligned}$$

The payoffs of the two players, when first channel is depolarizing and second channel is phase-damping, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^{DP} \$_{00} + \chi^{DP} \$_{11} + \Delta_{\mu 1}^4 (\$_{01} + \$_{10}) + (\$_{00} - \$_{11}) \Delta_{\mu 1}^3 \mu_p^2 \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\chi^{DP} \$_{00} + \eta^{DP} \$_{11} + \Delta_{\mu 1}^4 (\$_{01} + \$_{10}) - (\$_{00} - \$_{11}) \Delta_{\mu 1}^3 \mu_p^2 \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\chi^{DP} \$_{01} + \eta^{DP} \$_{10} + \Delta_{\mu 1}^4 (\$_{00} + \$_{11}) + (\$_{01} - \$_{10}) \Delta_{\mu 1}^3 \mu_p^2 \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\chi^{DP} \$_{10} + \eta^{DP} \$_{01} + \Delta_{\mu 1}^4 (\$_{00} + \$_{11}) - (\$_{01} - \$_{10}) \Delta_{\mu 1}^3 \mu_p^2 \xi \cos 2(\alpha_1 - \beta_2)] \\
& - (\frac{1}{4} \Delta_{\mu 1}^3) (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& - (\frac{1}{4} \eta_{1DP} \mu_p^2) (\$_{00} - \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + (\frac{1}{4} \Delta_{\mu 1}^3) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + (\frac{1}{4} \eta_{1DP} \mu_p^2) (\$_{01} - \$_{10}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)] \quad (29)
\end{aligned}$$

The payoffs of the two players, when first channel is phase-damping and second channel is depolarizing, are given by

$$\begin{aligned}
\$(\theta_i, \alpha_i, \beta_i) = & c_1 c_2 [\eta^{PD} \$_{00} + \chi^{PD} \$_{11} + \Delta_{\mu 2}^2 (\$_{01} + \$_{10}) + (\$_{00} - \$_{11})(\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \mu_p^1 \xi \cos 2(\alpha_1 + \alpha_2)] \\
& + s_1 s_2 [\chi^{PD} \$_{00} + \eta^{PD} \$_{11} + \Delta_{\mu 2}^2 (\$_{01} + \$_{10}) - (\$_{00} - \$_{11})(\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \mu_p^1 \xi \cos 2(\beta_1 + \beta_2)] \\
& + s_1 c_2 [\chi^{PD} \$_{01} + \eta^{PD} \$_{10} + \Delta_{\mu 2}^1 (\$_{00} + \$_{11}) + (\$_{01} - \$_{10})(\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \mu_p^1 \xi \cos 2(\alpha_2 - \beta_1)] \\
& + c_1 s_2 [\chi^{PD} \$_{10} + \eta^{PD} \$_{01} + \Delta_{\mu 2}^1 (\$_{00} + \$_{11}) - (\$_{01} - \$_{10})(\Delta_{\mu 2}^4 - \frac{2}{3} \mu_2 p_2) \mu_p^1 \xi \cos 2(\alpha_1 - \beta_2)] \\
& - (\frac{1}{4} \mu_p^1 \Delta_{\mu 2}^1 - \frac{1}{2} \mu_p^1 \Delta_{\mu 2}^2 + \frac{1}{4} \mu_p^1 \Delta_{\mu 2}^3) (\$_{00} + \$_{11}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + (\frac{1}{4} \Delta_{\mu 2}^4 - \frac{1}{6} \mu_p^1 \mu_2 p_2) (\$_{00} - \$_{11}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \\
& + (\frac{1}{4} \mu_p^1 \Delta_{\mu 2}^1 - \frac{1}{2} \mu_p^1 \Delta_{\mu 2}^2 + \frac{1}{4} \mu_p^1 \Delta_{\mu 2}^3) (\$_{01} + \$_{10}) \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 + \alpha_2 - \beta_1 - \beta_2) \\
& + (\frac{1}{4} \Delta_{\mu 2}^4 - \frac{1}{6} \mu_p^1 \mu_2 p_2) (\$_{01} - \$_{10}) \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\alpha_1 - \alpha_2 + \beta_1 - \beta_2)] \quad (30)
\end{aligned}$$

The definitions of the parameters in the payoffs for equations (22) to (30) are given in appendix B.

The payoff for the two players can be found by substituting the appropriate values for $\$_{ij}$ (elements of payoff matrix for the corresponding game as given in appendix A) in the above equations. These payoffs become the classical payoffs for $\gamma = \delta = 0$ and $p_1 = p_2 = 0$. It can be easily proved that for $\gamma = \delta = \pi/2$, with $\beta_1 = \beta_2 = 0$, $\mu_1 = \mu_2 = 0$ and $p_1 = 1$ or $p_2 = 1$, the results of ref. [17] are reproduced for all the nine cases in Prisoner's Dilemma game. Nawaz and Toor have shown that in case of phase-damping channel, for maximum correlation the effects of decoherence diminish and it behaves as a noiseless game [22]. However, in case of amplitude and depolarizing channels, for maximum correlation the effects of decoherence persist and causes a reduction in the payoffs and it does not behave as a noiseless game.

III. RESULTS AND DISCUSSIONS

To analyze the effects of memory in quantum games, we consider a situation in which Alice is restricted to play classical strategies, i.e., $\alpha_1 = \beta_1 = 0$, whereas Bob is allowed to play the quantum strategies as well. Under these circumstances following four cases for the different combinations of δ and γ are worth noting;

Case(i): When $\delta = \gamma = 0$, the payoffs reduce to classical results for unital case i.e. phase-damping and depolarizing channels. These payoffs, as expected, are independent of the quantum strategies α_2, β_2 , but only depend upon decoherence parameter p and the memory parameter μ . For non-unital case, i.e. amplitude-damping channel, the results reduce to classical game when we put

$p_1 = p_2 = 0$ along with $\mu_1 = \mu_2 = 0$. However, the payoffs of the two players remain independent of quantum phases and the decrease due to decoherence is compensated by the memory and payoffs are enhanced from their classical counterparts (which can be seen from figure 2 for all the three games).

Case(ii): When $\delta = 0$, $\gamma \neq 0$, and channels 1 & 2 are amplitude-damping;

a) In case of Prisoner's Dilemma and Chicken games, the effect of memory can be summarized as; when p increases the payoffs start decreasing, however, this effect is partially overcome by the addition of memory i.e. as μ increases the payoff increases and as result it compensates the reduction in player's payoffs due to decoherence (as shown in figure 3).

b) In case of Battle of Sexes game, the quantum player enjoys an advantage over classical player for $0 < p \leq 1$ (it can be seen from figure 4). The optimal strategy for Bob is to play $\alpha_2 = \pi/2$ and $\beta_2 = 0$.

c) When channels 1 & 2 are phase-damping and amplitude-damping or depolarizing and amplitude-damping respectively, the quantum player remains superior over the classical player in case of Battle of Sexes game only (which can be seen from figure 4).

d) When channels 1 & 2 are phase-damping or depolarizing, the payoffs of the players remains equal in all the three games, however, memory controls the payoffs reduction due to decoherence.

Case(iii): When $\gamma = 0$, $\delta \neq 0$, and channels 1 & 2 are depolarizing,

a) In case of Battle of Sexes game, the quantum player outperforms the classical player for $0 < p \leq 1$ (it can be seen from figure 5). The optimal strategy for Bob is to play $\alpha_2 = 0$ and $\beta_2 = \pi/2$.

b) For phase-damping or amplitude-damping channels, the payoffs of the players remain equal in all the three games considered and memory compensates the decoherence effects in Prisoner's Dilemma, and Chicken games.

c) It can be seen from figure 5 that when channels 1 & 2 are phase-damping and amplitude-damping or depolarizing and amplitude-damping respectively, the quantum player remains superior over the classical player in case of Battle of Sexes game.

Case(iv): when $\gamma = \delta = \pi/2$, with $\mu = 0$ (memoryless case), the quantum player is better off for $p < 1$ for all the three channels. For $\mu \neq 0$, the quantum player outperform classical player even for maximum noise, i.e., $p = 1$, for all the nine cases, which is not possible in memoryless case (it can be seen from figures 6 and 7) for amplitude-damping and depolarizing channels, the similar behaviour is seen for all the remaining 7 channels).

A Nash equilibrium implies that no player can increase his/her payoff by unilaterally changing

his/her strategy. One can see from case (ii)-b that for Alice $\theta_1 = 0$ and for Bob $\theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ remain their best strategies throughout the course of the game for the entire range of the decoherence parameter p and the memory parameter μ . Similarly, for case (iii)-a, it can be seen that for Alice $\theta_1 = 0$ and for Bob $\theta_2 = \pi/2$ and $\alpha_2 = 0$, $\beta_2 = \pi/2$ remain their best strategies for all values of p and μ and no player can increase his/her payoff by unilaterally changing his/her strategy. A similar situation occurs for all the remaining cases. Thus by inspection (from equations (22) to (30)), one can see that the Nash equilibria of the three games do not change under the effect of quantum memory.

IV. CONCLUSIONS

Quantum games with correlated noise are studied under the generalized quantization scheme [24]. Three games, Prisoner's Dilemma, Battle of Sexes and Chicken are studied with one player restricted to the classical strategies while the other is allowed to play quantum strategies. It is shown that the effects of the memory and decoherence become effective for the case $\gamma = \delta = \pi/2$, for which the quantum player out performs the classical player in all the three games for maximally entangled case. It is also shown that the quantum player enjoys an advantage over classical player for $\delta = 0$, $\gamma \neq 0$ and $\gamma = 0$, $\delta \neq 0$ cases in Battle of Sexes game when amplitude-damping and depolarizing channels are used respectively. It can be seen that the Nash equilibria of the three games do not change under the effect of memory.

Appendix A: Classical Games

Brief descriptions of three classical games, Prisoner's Dilemma, Battle of Sexes and Chicken are given below,

Prisoner's Dilemma

This game depicts a situation where two prisoners, who have committed a crime together, are being interrogated in separate cells. The two possible moves for each prisoner are, to cooperate (C) or to defect (D). They are not allowed to communicate but have access to the following payoff matrix:

$$\begin{array}{cc}
 & \text{Bob} \\
 & C \quad D \\
 \text{Alice} \begin{array}{c} C \\ D \end{array} & \left[\begin{array}{cc} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{array} \right]
 \end{array} \tag{A1}$$

It is clear from the payoff matrix A1 that D is the dominant strategy for the two players. Therefore, rational reasoning forces the players to play D . Hence (D, D) is the Nash equilibrium of the game with payoffs $(1, 1)$. But the players could get higher payoffs if they would have played C instead of D . This is the dilemma of the game.

Battle of Sexes

The payoff matrix for Battle of Sexes game is

$$\begin{array}{cc}
 & \text{Bob} \\
 & O \quad T \\
 \text{Alice} \begin{array}{c} O \\ T \end{array} & \left[\begin{array}{cc} (2, 1) & (0, 0) \\ (0, 0) & (1, 2) \end{array} \right]
 \end{array} \tag{A2}$$

In this game Alice is fond of Opera whereas Bob likes watching TV but they also want to spend the evening together. The two pure Nash equilibria (NE) of this game are (O, O) and (T, T) which corresponds to the situation when both the players choose Opera and TV, respectively. Here the first NE is more favorable to Alice while the second NE is favorable to Bob. Since they are not allowed to communicate, So, they face a dilemma in choosing their strategies.

The Chicken game

The payoff matrix for the Chicken game is

$$\begin{array}{cc}
 & \text{Bob} \\
 & C \quad D \\
 \text{Alice} \begin{array}{c} C \\ D \end{array} & \left[\begin{array}{cc} (3, 3) & (1, 4) \\ (4, 1) & (0, 0) \end{array} \right]
 \end{array} \tag{A3}$$

In the game of Chicken, also known as the Hawk-Dove game, two players drove their cars towards each other. The first one to swerve to avoid collision is the loser (chicken) and the one who keeps

on driving straight is the winner. There is no dominant strategy in this game. There are two NE (C, D) and (D, C) , the former is preferred by Bob and the latter is preferred by Alice. The dilemma of this game is that the Pareto Optimal strategy (C, C) is not Nash equilibrium.

Appendix B: Some Definitions

The definitions of the parameters used in equation (22) are given as

$$\begin{aligned}\eta_1^A &= \chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(00)} \cos^2(\gamma/2) \cos^2(\delta/2) + (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2) \\ &\quad + (\chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(11)} + 2\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(a)}) \sin^2(\delta/2) \cos^2(\gamma/2)\end{aligned}$$

$$\begin{aligned}\eta_2^A &= \chi_{\mu_2}^{(00)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \cos^2(\delta/2) + (\chi_{\mu_1}^{(00)} + 2\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(a)}) \times \\ &\quad \sin^2(\delta/2) \cos^2(\gamma/2) + \chi_{\mu_1}^{(11)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2)\end{aligned}$$

$$\begin{aligned}\eta_3^A &= \chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(00)} \cos^2(\gamma/2) \cos^2(\delta/2) + (\chi_{\mu_1}^{(01)} + \chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(11)}) \cos^2(\gamma/2) \sin^2(\delta/2) \\ &\quad + \chi_{\mu_2}^{(a)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2) + \chi_{\mu_1}^{(00)} \cos^2(\gamma/2)) \sin^2(\delta/2)\end{aligned}$$

$$\eta_4^A = \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(00)} \cos^2(\delta/2) + (\chi_{\mu_1}^{(10)} + \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(11)} - 2\chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(a)}) \sin^2(\delta/2)$$

$$\begin{aligned}\chi_1^A &= \chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(00)} \cos^2(\gamma/2) \sin^2(\delta/2) + (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \cos^2(\delta/2) \\ &\quad + (\chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(11)} + 2\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(a)}) \cos^2(\delta/2) \cos^2(\gamma/2)\end{aligned}$$

$$\begin{aligned}\chi_2^A &= \chi_{\mu_2}^{(00)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2) + (\chi_{\mu_1}^{(00)} + 2\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(a)}) \times \\ &\quad \cos^2(\delta/2) \cos^2(\gamma/2) + \chi_{\mu_2}^{(11)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \cos^2(\delta/2)\end{aligned}$$

$$\begin{aligned}\chi_3^A &= (\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(00)} + \chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(11)}) \cos^2(\gamma/2) \sin^2(\delta/2) + \chi_{\mu_1}^{(01)} \cos^2(\gamma/2) \cos^2(\delta/2) \\ &\quad + \chi_{\mu_2}^{(a)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2) + \chi_{\mu_1}^{(00)} \cos^2(\gamma/2)) \cos^2(\delta/2)\end{aligned}$$

$$\chi_4^A = \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(00)} \sin^2(\delta/2) + (\chi_{\mu_1}^{(10)} + \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(11)} - 2\chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(a)}) \cos^2(\delta/2)$$

$$\Delta_1^A = (\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(b)} + \chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(01)}) \cos^2(\gamma/2)$$

$$\Delta_2^A = \chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(b)} \cos^2(\gamma/2) + \chi_{\mu_2}^{(01)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2))$$

$$\begin{aligned}\Delta_3^A &= \chi_{\mu_2}^{(b)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \cos^2(\delta/2) \\ &\quad + (\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(01)} + \chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(b)} \sin^2(\delta/2)) \cos^2(\gamma/2)\end{aligned}$$

$$\begin{aligned}\Delta_4^A &= \chi_{\mu_2}^{(b)} (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2) \\ &\quad + (\chi_{\mu_1}^{(01)} \chi_{\mu_2}^{(01)} + \chi_{\mu_1}^{(00)} \chi_{\mu_2}^{(b)} \cos^2(\delta/2)) \cos^2(\gamma/2)\end{aligned}$$

$$\Delta_5^A = (\sin^2(\gamma/2) + \chi_{\mu_1}^{(11)} \cos^2(\gamma/2)) + \chi_{\mu_1}^{(00)} - 2\chi_{\mu_1}^{(01)} \cos^2(\gamma/2),$$

$$\Delta_6^A = \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(01)} - \chi_{\mu_1}^{(10)} \chi_{\mu_2}^{(b)}, \quad \xi = \frac{1}{2} \sin(\delta) \sin(\gamma),$$

$$s_i = \sin^2\left(\frac{\theta_i}{2}\right), \quad c_i = \cos^2\left(\frac{\theta_i}{2}\right)$$

$$\begin{aligned}
\chi_{\mu_1}^{(00)} &= (1-p_1)^2 + \mu_1(1-p_1)p_1, \\
\chi_{\mu_2}^{(00)} &= (1-p_2)^2 + \mu_2(1-p_2)p_2 \\
\chi_{\mu_1}^{(11)} &= p_1^2 + \mu_1(1-p_1)p_1, \\
\chi_{\mu_2}^{(11)} &= p_2^2 + \mu_2(1-p_2)p_2 \\
\chi_{\mu_1}^{(10)} &= (1-\mu_1)(1-p_1) + \mu_1(1-p_1)^{\frac{1}{2}}, \\
\chi_{\mu_2}^{(10)} &= (1-\mu_2)(1-p_2) + \mu_2(1-p_2)^{\frac{1}{2}} \\
\chi_{\mu_1}^{(01)} &= (1-\mu_1)(1-p_1)p_1, \quad \chi_{\mu_2}^{(01)} = (1-\mu_2)(1-p_2)p_2 \\
\chi_{\mu_2}^{(a)} &= (1-\mu_2)p_2, \quad \chi_{\mu_2}^{(b)} = (1-p_2) + \mu_2p_2
\end{aligned}$$

The definitions of the parameters used in equation (23) are given as

$$\begin{aligned}
\Delta_{\mu_1}^1 &= -\frac{1}{9}(-3+2p_1)(-2p_1+2\mu_1p_1+3) \\
\Delta_{\mu_1}^2 &= -\frac{2}{9}p_1(-2p_1+2\mu_1p_1-3\mu_1) \\
\Delta_{\mu_1}^3 &= -\frac{1}{9}(-9+24p_1-18\mu_1p_1-16p_1^2+16\mu_1p_1^2)-\frac{2}{3}\mu_1p_1 \\
\Delta_{\mu_1}^4 &= \frac{2}{9}p_1(-3+2p_1)(\mu_1-1) \\
\Delta_{\mu_2}^1 &= -\frac{1}{9}(-3+2p_2)(-2p_2+2\mu_2p_2+3) \\
\Delta_{\mu_2}^2 &= \frac{2}{9}p_2(-3+2p_2)(\mu_2-1) \\
\Delta_{\mu_2}^3 &= -\frac{2}{9}p_2(-2p_2+2\mu_2p_2-3\mu_2) \\
\Delta_{\mu_2}^4 &= -\frac{1}{9}(-9+24p_2-18\mu_2p_2-16p_2^2+16\mu_2p_2^2) \\
\Delta_{\mu}^{11} &= \Delta_{\mu_1}^1 \cos^2(\gamma/2) + \Delta_{\mu_1}^2 \sin^2(\gamma/2) \\
\Delta_{\mu}^{21} &= \Delta_{\mu_1}^2 \cos^2(\gamma/2) + \Delta_{\mu_1}^1 \sin^2(\gamma/2) \\
\eta^D &= (\Delta_{\mu_2}^1 \Delta_{\mu}^{11} + \Delta_{\mu_2}^3 \Delta_{\mu}^{21}) \cos^2(\delta/2) + (\Delta_{\mu_2}^1 \Delta_{\mu}^{21} + \Delta_{\mu_2}^3 \Delta_{\mu}^{11}) \sin^2(\delta/2) + 2\Delta_{\mu_2}^2 \Delta_{\mu_1}^4 \\
\chi^D &= (\Delta_{\mu_2}^1 \Delta_{\mu}^{21} + \Delta_{\mu_2}^3 \Delta_{\mu}^{11}) \cos^2(\delta/2) + (\Delta_{\mu_2}^1 \Delta_{\mu}^{11} + \Delta_{\mu_2}^3 \Delta_{\mu}^{21}) \sin^2(\delta/2) + 2\Delta_{\mu_2}^2 \Delta_{\mu_1}^4 \\
\Delta^D &= \Delta_{\mu_2}^2 \Delta_{\mu}^{11} + \Delta_{\mu_2}^2 \Delta_{\mu}^{21} + \Delta_{\mu_2}^1 \Delta_{\mu_1}^4 + \Delta_{\mu_2}^3 \Delta_{\mu_1}^4 \\
\eta_{1DP} &= -(\Delta_{\mu_1}^2 \cos^2(\gamma/2) + \Delta_{\mu_1}^1 \sin^2(\gamma/2)) - (\Delta_{\mu_1}^2 \sin^2(\gamma/2) + \Delta_{\mu_1}^1 \cos^2(\gamma/2)) + 2\Delta_{\mu_1}^4
\end{aligned}$$

The definitions of the parameters used in equation (24) are given as

$$\begin{aligned}
\eta^P &= \cos^2(\gamma/2) \cos^2(\delta/2) + \sin^2(\gamma/2) \sin^2(\delta/2) \\
\chi^P &= \sin^2(\gamma/2) \cos^2(\delta/2) + \cos^2(\gamma/2) \sin^2(\delta/2) \\
\mu_p^{(i)} &= (1-\mu_i)(1-p_i)^2 + \mu_i
\end{aligned}$$

The definitions of the parameters used in equation (25) are given as

$$\begin{aligned}
\eta_1^{PA} &= \chi_{\mu 2}^{(00)} \cos^2(\gamma/2) \cos^2(\delta/2) + (\sin^2(\gamma/2) + \chi_{\mu 2}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2) \\
\eta_2^{PA} &= (\cos^2(\gamma/2) + \chi_{\mu 2}^{(11)} \sin^2(\gamma/2)) \sin^2(\delta/2) + \chi_{\mu 2}^{(00)} \sin^2(\gamma/2) \cos^2(\delta/2) \\
\eta_3^{PA} &= \chi_{\mu 2}^{(a)} \sin^2(\gamma/2) \sin^2(\delta/2) + \chi_{\mu 2}^{(a)} \cos^2(\gamma/2) \sin^2(\delta/2) \\
\chi_1^{PA} &= (\cos^2(\gamma/2) + \chi_{\mu 2}^{(11)} \sin^2(\gamma/2)) \cos^2(\delta/2) + \chi_{\mu 2}^{(00)} \cos^2(\gamma/2) \sin^2(\delta/2) \\
\chi_2^{PA} &= \chi_{\mu 2}^{(00)} \sin^2(\gamma/2) \sin^2(\delta/2) + (\cos^2(\gamma/2) + \chi_{\mu 2}^{(11)} \sin^2(\gamma/2)) \cos^2(\delta/2) \\
\chi_3^{PA} &= \chi_{\mu 2}^{(a)} \sin^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 2}^{(a)} \cos^2(\gamma/2) \cos^2(\delta/2) \\
\Delta_1^{PA} &= \chi_{\mu 2}^{(01)} \cos^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 2}^{(01)} \cos^2(\gamma/2) \sin^2(\delta/2) \\
\Delta_2^{PA} &= \chi_{\mu 2}^{(01)} \sin^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 2}^{(01)} \sin^2(\gamma/2) \sin^2(\delta/2) \\
\Delta_3^{PA} &= \chi_{\mu 2}^{(b)} \sin^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 2}^{(b)} \cos^2(\gamma/2) \sin^2(\delta/2) \\
\Delta_4^{PA} &= \chi_{\mu 2}^{(b)} \cos^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 2}^{(b)} \sin^2(\gamma/2) \sin^2(\delta/2)
\end{aligned}$$

The definitions of the parameters used in equation (26) are given as

$$\begin{aligned}
\eta^{AP} &= \chi_{\mu 1}^{(00)} \cos^2(\gamma/2) \cos^2(\delta/2) + (\sin^2(\gamma/2) + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2)) \sin^2(\delta/2) \\
\chi^{AP} &= (\sin^2(\gamma/2) + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2)) \cos^2(\delta/2) + \chi_{\mu 1}^{(00)} \cos^2(\gamma/2) \sin^2(\delta/2) \\
\Delta^{AP} &= \chi_{\mu 1}^{(01)} \cos^2(\gamma/2) \cos^2(\delta/2) + \chi_{\mu 1}^{(01)} \cos^2(\gamma/2) \sin^2(\delta/2)
\end{aligned}$$

The definitions of the parameters used in equation (27) are given as

$$\begin{aligned}
\eta^{AD} &= (\chi_{\mu 1}^{(00)} \Delta_{\mu 2}^1 \cos^2(\gamma/2) + \Delta_{\mu 2}^3 (\sin^2(\gamma/2) + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2))) \cos^2(\delta/2) + (\Delta_{\mu 2}^1 (\sin^2(\gamma/2) \\
&\quad + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2)) + \chi_{\mu 1}^{(00)} \Delta_{\mu 2}^3 \cos^2(\gamma/2)) \sin^2(\delta/2) + 2\chi_{\mu 1}^{(01)} \Delta_{\mu 2}^2 \cos^2(\gamma/2) \\
\chi^{AD} &= (\chi_{\mu 1}^{(00)} \Delta_{\mu 2}^1 \cos^2(\gamma/2) + \Delta_{\mu 2}^3 (\sin^2(\gamma/2) + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2))) \sin^2(\delta/2) + (\Delta_{\mu 2}^1 (\sin^2(\gamma/2) \\
&\quad + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2)) + \chi_{\mu 1}^{(00)} \Delta_{\mu 2}^3 \cos^2(\gamma/2)) \cos^2(\delta/2) + 2\chi_{\mu 1}^{(01)} \Delta_{\mu 2}^2 \cos^2(\gamma/2) \\
\Delta^{AD} &= \chi_{\mu 1}^{(01)} (\Delta_{\mu 2}^1 + \Delta_{\mu 2}^3) \cos^2(\gamma/2) + \Delta_{\mu 2}^2 (\sin^2(\gamma/2) + \chi_{\mu 1}^{(11)} \cos^2(\gamma/2)) + \chi_{\mu 1}^{(00)} \Delta_{\mu 2}^2 \cos^2(\gamma/2) \\
\eta_{1AD} &= \chi_{\mu 1}^{(11)} \cos^2(\gamma/2) + \chi_{\mu 1}^{(00)} \cos^2(\gamma/2) - 2\chi_{\mu 1}^{(01)} \cos^2(\gamma/2) + \sin^2(\gamma/2)
\end{aligned}$$

The definitions of the parameters used in equation (28) are given as

$$\begin{aligned}
\eta_1^{DA} &= \Delta_\mu^{11} \chi_{\mu 2}^{(00)} \cos^2(\delta/2) + \Delta_\mu^{11} \chi_{\mu 2}^{(11)} \sin^2(\delta/2) + \Delta_\mu^{12} \sin^2(\delta/2) + 2\Delta_{\mu 1}^4 \chi_{\mu 2}^{(a)} \sin^2(\delta/2) \\
\eta_2^{DA} &= \Delta_\mu^{21} \chi_{\mu 2}^{(00)} \cos^2(\delta/2) + \Delta_\mu^{21} \chi_{\mu 2}^{(11)} \sin^2(\delta/2) + \Delta_\mu^{11} \sin^2(\delta/2) + 2\Delta_{\mu 1}^4 \chi_{\mu 2}^{(a)} \sin^2(\delta/2) \\
\eta_3^{DA} &= (\Delta_\mu^{11} \chi_{\mu 2}^{(a)} + \Delta_\mu^{21} \chi_{\mu 2}^{(a)}) \sin^2(\delta/2) + \Delta_{\mu 1}^4 \chi_{\mu 2}^{(00)} \cos^2(\delta/2) + (\chi_{\mu 2}^{(11)} + 1) \Delta_{\mu 1}^4 \sin^2(\delta/2) \\
\chi_1^{DA} &= \Delta_\mu^{11} \chi_{\mu 2}^{(00)} \sin^2(\delta/2) + \Delta_\mu^{11} \chi_{\mu 2}^{(11)} \cos^2(\delta/2) + \Delta_\mu^{12} \cos^2(\delta/2) + 2\Delta_{\mu 1}^4 \chi_{\mu 2}^{(a)} \cos^2(\delta/2) \\
\chi_2^{DA} &= \Delta_\mu^{21} \chi_{\mu 2}^{(00)} \sin^2(\delta/2) + \Delta_\mu^{21} \chi_{\mu 2}^{(11)} \cos^2(\delta/2) + \Delta_\mu^{11} \cos^2(\delta/2) + 2\Delta_{\mu 1}^4 \chi_{\mu 2}^{(a)} \cos^2(\delta/2) \\
\chi_3^{DA} &= (\Delta_\mu^{11} \chi_{\mu 2}^{(a)} + \Delta_\mu^{21} \chi_{\mu 2}^{(a)}) \cos^2(\delta/2) + \Delta_{\mu 1}^4 \chi_{\mu 2}^{(00)} \sin^2(\delta/2) + (\chi_{\mu 2}^{(11)} + 1) \Delta_{\mu 1}^4 \cos^2(\delta/2) \\
\Delta_1^{DA} &= \Delta_\mu^{11} \chi_{\mu 2}^{(01)} + \Delta_{\mu 1}^4 \chi_{\mu 2}^{(b)} \\
\Delta_2^{DA} &= \Delta_\mu^{11} \chi_{\mu 2}^{(b)} \sin^2(\delta/2) + \Delta_\mu^{21} \chi_{\mu 2}^{(b)} \cos^2(\delta/2) + \Delta_{\mu 1}^4 \chi_{\mu 2}^{(01)} \\
\Delta_3^{DA} &= \Delta_\mu^{11} \chi_{\mu 2}^{(b)} \cos^2(\delta/2) + \Delta_\mu^{21} \chi_{\mu 2}^{(b)} \sin^2(\delta/2) + \Delta_{\mu 1}^4 \chi_{\mu 2}^{(01)}
\end{aligned}$$

The definitions of the parameters used in equation (29) are given as

$$\begin{aligned}
\eta^{DP} &= \Delta_\mu^{11} \cos^2(\delta/2) + \Delta_\mu^{12} \sin^2(\delta/2) \\
\chi^{DP} &= \Delta_\mu^{11} \sin^2(\delta/2) + \Delta_\mu^{12} \cos^2(\delta/2)
\end{aligned}$$

The definitions of the parameters used in equation (30) are given as

$$\begin{aligned}
\eta^{PD} &= (\Delta_{\mu 2}^1 \cos^2(\gamma/2) + \Delta_{\mu 2}^3 \sin^2(\gamma/2)) \cos^2(\delta/2) \\
&\quad + (\Delta_{\mu 2}^1 \sin^2(\gamma/2) + \Delta_{\mu 2}^3 \cos^2(\gamma/2)) \sin^2(\delta/2) \\
\chi^{PD} &= (\Delta_{\mu 2}^1 \sin^2(\gamma/2) + \Delta_{\mu 2}^3 \cos^2(\gamma/2)) \cos^2(\delta/2) \\
&\quad + (\Delta_{\mu 2}^1 \cos^2(\gamma/2) + \Delta_{\mu 2}^3 \sin^2(\gamma/2)) \sin^2(\delta/2)
\end{aligned}$$

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- [1] Eisert J, Wilkins M and Lewenstein M 1999 Phys. Rev. Lett. **83** 3077.
 - [2] Flitney A P and Abbott D 2002 Fluct. Noise Lett. **2** R175.
 - [3] Lee C F and Johnson N 2002 Phys. Rev. A **67** 022311.
 - [4] Du J, Li H et al 2003 J. Phys. A **36** 6551.
 - [5] Benjamin S C and Hayden P M 2001 Phys. Rev. A **64** 030301(R).
 - [6] Nawaz A and Toor A H 2004 J. Phys. A: Math. Gen. **37** 4437.
 - [7] Eisert J and Wilkins M 2000 J. Mod. Opt. **47** 2543.
 - [8] Iqbal A and Toor A H 2000 Phys. Lett. A **280** 249.
 - [9] Kay R, Johnson N F, Benjamin S C 2001 J. Phys. A **34** L547.

- [10] Flitney A P, Abbott D 2002 Phys. Rev. A **65** 062318.
- [11] Marinatto L and Weber T 2000 Phys. Lett. A **272** 291.
- [12] Du J, Li H, Xu X, Shi M, Wu J, Zhou X, Han R 2002 Phys. Rev. Lett. **88** 137902.
- [13] Lee C F and Johnson N 2002 Phys. World **15** 25.
- [14] Klarreich E 2001 Nature **414** 244.
- [15] Abbott D, Davies P C W and Shalizi C R 2002 Fluct. Noise Lett. **2** C1-12.
- [16] Meyer D A 1999 Phys. Rev. Lett. **82** 1052.
- [17] Chen K L, Ang H, Kiang D, Kwek L C and Lo C F 2003 Phys. Lett. A **316** 317.
- [18] Macchiavello C and Palma G M 2002 Phys. Rev. A **65** 050301.
- [19] Yeo Ye and Skeen Andrew 2003 Phys. Rev. A **67** 064301.
- [20] Lidar D A and Whaley K B 2003 Springer Lecturer Notes in Physics vol **622** ed Benatti F and Floreanini R (Berlin: Springer) pp 83-120.
- [21] Preskill J 1998 Proc. R. Soc. Lond. A **454** 385.
- [22] Nawaz A and Toor A H 2006 J. Phys. A: Math. Gen. **39** 9321.
- [23] Cao S, Fang M et al 2007 Chin. Phys. **16** 915.
- [24] Nawaz A and Toor A H 2004 J. Phys. A: Math. Gen. **37** 11457.
- [25] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press).
- [26] Karpov E, Daems D and Cerf N J 2006 Phys. Rev. A **74** 032320.
- [27] Flitney A P and Abbott D 2005 J. Phys. A **38** 449.

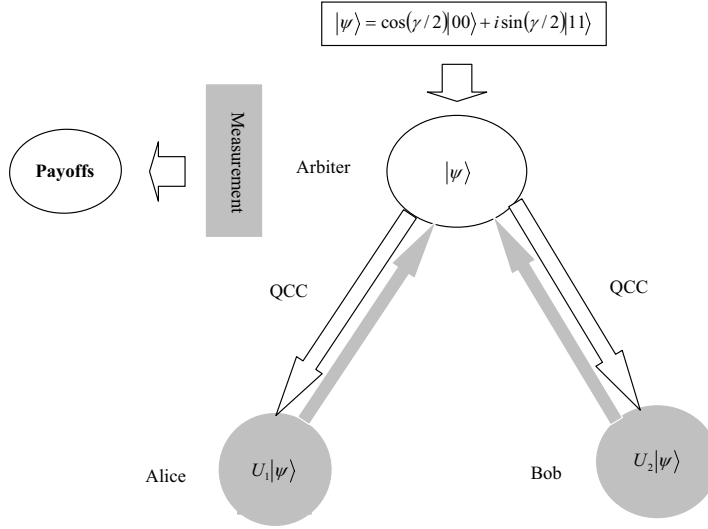


FIG. 1: Schematic diagram of the model.

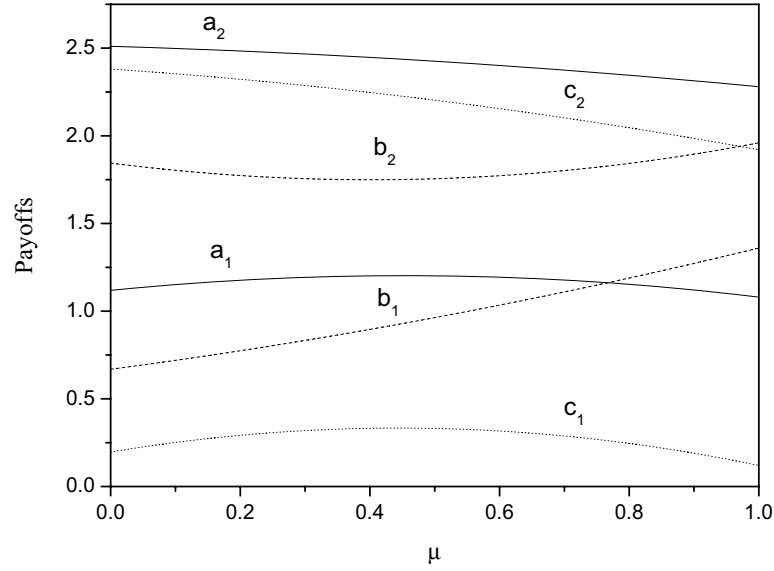


FIG. 2: Players (Alice/Bob) payoffs as a function of the memory parameter, μ is plotted for the quantum games Prisoner's Dilemma (solid lines, a_i), Battle of the Sexes (dashed lines, b_i) and Chicken (dotted lines, c_i) for amplitude-damping channel. Indices 1 & 2 correspond to $p = 0.8$ & $p = 0.2$ respectively with $\delta = \gamma = 0$, $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ as Bob's optimal strategy.

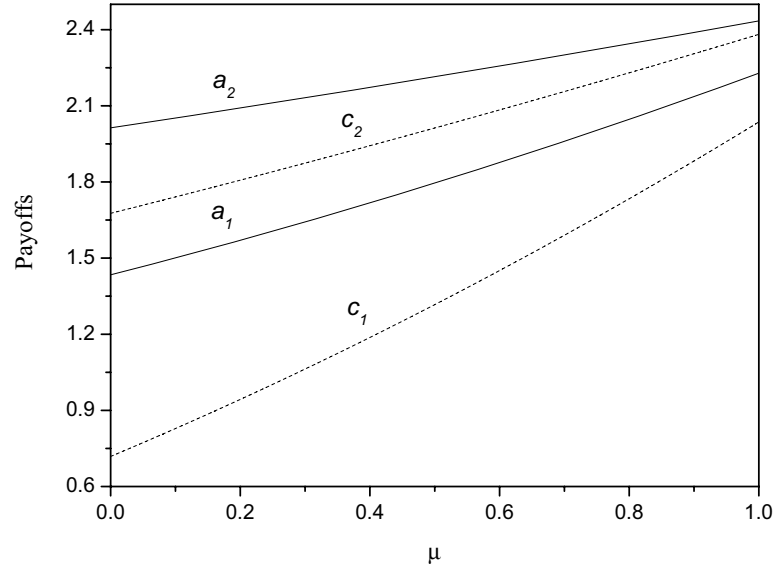


FIG. 3: Bob's payoff as a function of memory parameter μ is plotted for Prisoner's Dilemma (solid lines, a_i) and Chicken (dashed lines, c_i) for amplitude-damping channel. Indices 1 & 2 correspond to $p = 0.8$ & $p = 0.2$ respectively with $\delta = 0$, $\gamma = \pi/2$, $\theta_1 = \theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ as his optimal strategy.

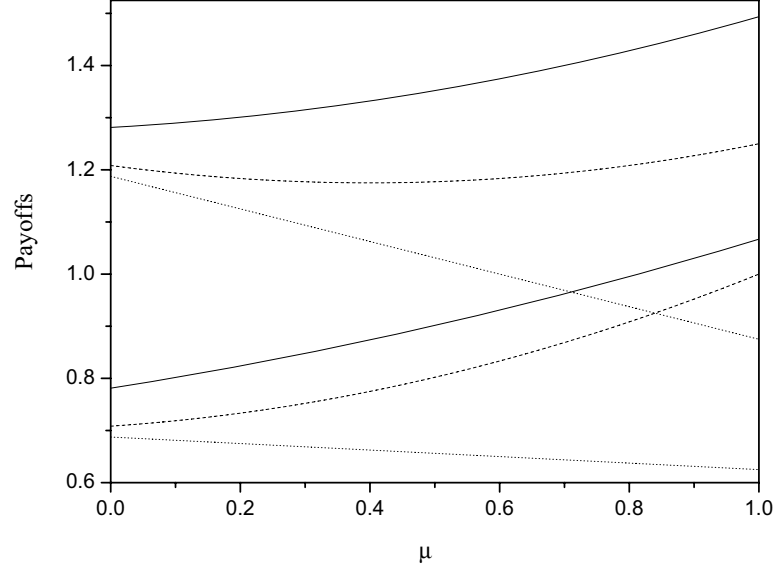


FIG. 4: Payoffs for Alice (classical player) and Bob (quantum player) are plotted as a function of memory parameter μ for amplitude-damping (solid lines), depolarizing followed by an amplitude-damping (dashed lines) and phase-damping followed by an amplitude-damping (dotted lines) channels for Battle of the Sexes game with $\delta = 0$, $\gamma = \pi/2$, $p = 0.5$, $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ as Bob's optimal strategy. The lower curves for all the three cases correspond to Alice's payoff.

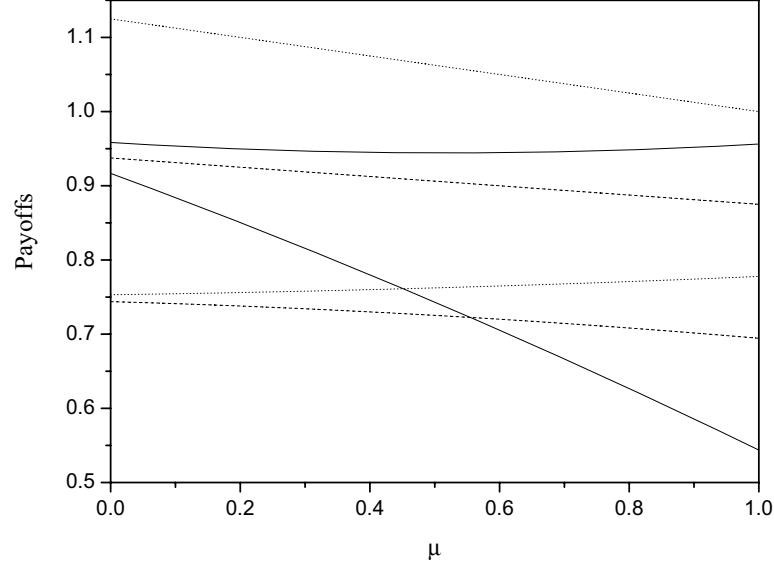


FIG. 5: Payoffs for Alice and Bob are plotted as a function of memory parameter μ for amplitude-damping (solid lines), depolarizing followed by an amplitude-damping (dashed lines) and phase-damping followed by an amplitude-damping (dotted lines) channels for Battle of the Sexes game with $\gamma = 0$, $\delta = \pi/2$, $p = 0.5$, $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\alpha_2 = 0$, $\beta_2 = \pi/2$ as Bob's optimal strategy. The lower curves for all the three cases correspond to Alice's payoff.

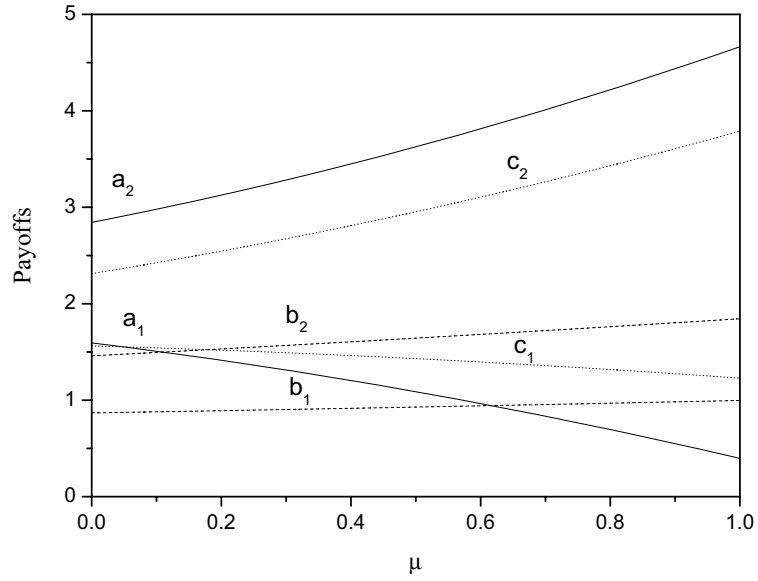


FIG. 6: Alice's (1) and Bob's (2) payoffs are plotted as a function of memory parameter μ for the quantum games Prisoner's Dilemma (solid lines, a_i), Battle of the Sexes (dashed lines, b_i) and Chicken (dotted lines, c_i) for amplitude-damping channel with $\delta = \gamma = \pi/2$, $p = 0.5$, $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ as Bob's optimal strategy.

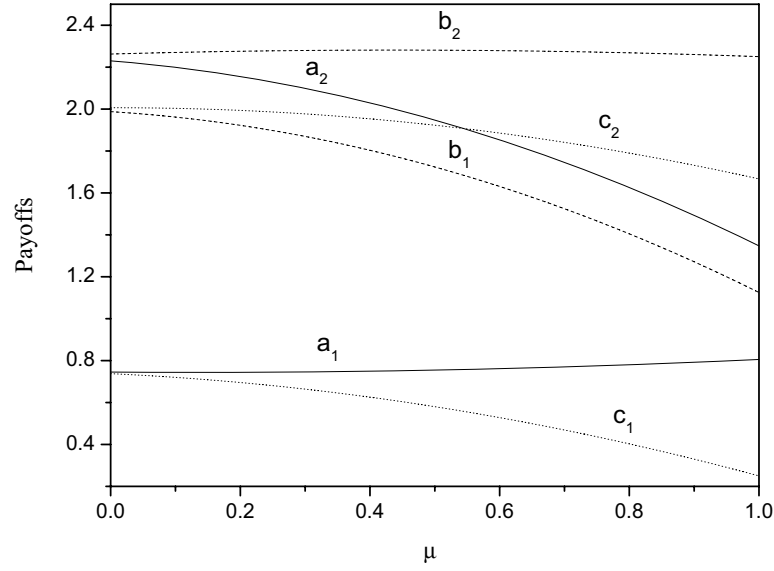


FIG. 7: Alice's (1) and Bob's (2) payoffs are plotted as a function of the memory parameter μ for the quantum games Prisoner's Dilemma (solid lines, a_i), Battle of the Sexes (dashed lines, b_i) and Chicken (dotted lines, c_i) for depolarizing channel with $\delta = \gamma = \pi/2$, $p = 0.5$, $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\alpha_2 = \pi/2$, $\beta_2 = 0$ as Bob's optimal strategy.